Enrollment No: ______________________ Exam Seat No: _____________________

C.U.SHAH UNIVERSITY

Winter Examination-2022

Subject Name: Real Analysis-I

Subject Code: 4SC05REA1 **Branch: B.Sc. (Mathematics)**

Semester : 5 **Date : 25/11/2022** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1	**Attempt the following questions:**		**[14]**
a)	Define: Null Set.		(02)
b)	State Raabe's test for series.		(02)
c)	Prove : $\|x\|^2 = x^2 = \|-x\|^2$,$\forall\, x \in R$.		(02)
d)	Define: Least Upper Bound of a sequence. Find lub of $\left\{\frac{n}{n+1} : n \in N\right\}$.		(02)
e)	Find the range set of the sequence $\{n^2 : n \in N\}$.		(02)
f)	Define: Bounded sequence.		(01)
g)	Define : Exponential Function.		(01)
h)	True/False : $\underline{A_n} \leq \underline{lim} a_n$ for all $n \in N$.		(01)
i)	True/False: $\sum \frac{1}{n^9}$ is divergent.		(01)

Attempt any four questions from Q-2 to Q-8

Q-2	**Attempt all questions**	**[14]**
a)	Prove that the set of Rational numbers is not ordered complete.	(07)
b)	Prove : (i) $\|x + y\| \leq \|x\| + \|y\|$ for $\forall\, x, y \in R$ (ii) $\|x - y\| \geq \|\|x\| - \|y\|\|$ for $\forall\, x, y \in R$	(05)
c)	Define : Conditionally Convergent Series and Absolutely Convergent Series.	(02)

Q-3	**Attempt all questions**	**[14]**
a)	Show that for any two positive real numbers there exist a positive integer n such that $na > b$.	(05)
b)	Prove : A sequence can't converge to more than one point .	(05)
c)	Prove that $\sin x$ is uniformly continuous on $[0, \infty)$.	(04)

Q-4	**Attempt all questions**	**[14]**
a)	State and prove Bolzano Weierstrass theorem for sequences.	(07)

b) Find $\overline{lim}a_n$ and $\underline{lim}a_n$ where $a_n = (-1)^n(1+\frac{1}{n})$. (05)

c) Prove that $\lim_{n\to\infty}\frac{1+3+5+....+(2n-1)}{n^2} = n$ (02)

Q-5 Attempt all questions **[14]**

a) (05)

Find the right hand and left hand limits of a function defined as follows:

$$f(x) = \begin{cases} \frac{|x-4|}{x-4} & ; \ x \neq 4 \\ 0 & ; \ x = 4 \end{cases}.$$

b) Prove : $\lim_{n\to\infty}\frac{3+2\sqrt{n}}{\sqrt{n}} = 2$ using definition. (05)

c) Find the limit point of $\left\{\frac{1}{n^2} : n \in N\right\}$ and $\left\{(-1)^n : n \in N\right\}$ if exists. (04)

Q-6 Attempt all questions **[14]**

a) Show that the sequence $\{r^n\}$ converges if $-1 < r \leq 1$. (07)

b) State and prove D'Alembert's ratio test. (07)

Q-7 Attempt all questions **[14]**

a) State and prove Cauchy's general principle of convergence for sequence. (07)

b) Show that the series $\sum_{n=1}^{\infty}\frac{1}{n^{1/p}-1}$ is convergent for $p < \frac{1}{2}$ and divergent for $p \geq \frac{1}{2}$. (05)

c) Test the convergence of the series $\sum_{n=1}^{\infty}\left(-\frac{5}{6}\right)^n$. (02)

Q-8 Attempt all questions **[14]**

a) State and prove Sandwich theorem for a sequence and using sandwich theorem prove $\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \dots + \frac{1}{\sqrt{n^2+n}}\right] = 1$. (06)

b) Show that a series $\frac{1\cdot 2}{3^2\cdot 4^2} + \frac{3\cdot 4}{5^2\cdot 6^2} + \frac{5\cdot 6}{7^2\cdot 8^2} + \cdots$ is convergent. (05)

c) Test the convergence of the series $\sum_{n=1}^{\infty}\frac{n+1}{n^p}$. (03)